Review: Recursive Function Theory

1. Evaluate each of the following expressions. In each case, your answer should be a natural number. Show your work.
   Assume that plus(x, y) = \text{def} x + y and mult(x, y) = \text{def} x \cdot y.
   a) Comp[plus, C_{20}, Comp[succ, C_{10}^5]] (10, 11, 12, 13, 14)
   b) Pr[p^1_1, Comp[mult, C_{10}^3, C_{10}^3]] (4, 2)

2. Show that each of the following functions is primitive recursive by writing it in canonical form (that is, in terms of succ, constant-0 functions, projection functions, Comp, Pr, and functions previously shown to be primitive recursive). You may assume that the following functions and predicates have been shown to be primitive recursive.

   \begin{align*}
   \text{plus}(x, y) &= \text{def} x + y \\
   \text{mult}(x, y) &= \text{def} x \cdot y \\
   \text{monus}(x, y) &= \text{def} x \cdot y \\
   \text{sg}(n) &= \text{def} x \geq y \\
   \text{sg}(n) &= \text{def} x \leq y \\
   \text{all the constant functions} &= \text{def} x \neq y \\
   \text{all the projection functions} &= \text{def} x \neq y \\
   \text{succ}(x) &= \text{def} x + 1 \\
   \text{pred}(x) &= \text{def} x - 1 \\
   \text{the characteristic functions} &= \chi_\text{x}(x, y), \chi_\text{y}(x, y), \\
   \chi_\text{Z}(x, y), \chi_\text{Z}(x, y), \chi_\text{Z}(x, y), \chi_\text{Z}(x, y)
   \end{align*}

   Do not assume that any other functions have been shown to be primitive recursive.
   a) h(x, y) = 8(x + y)
b) \[ h(x) = \begin{cases} 
4 & \text{if } x = 4 \\
0 & \text{otherwise}
\end{cases} \]

c) \[ h(x, y) = \begin{cases} 
5 & \text{if } y = 0 \\
(y - 1) + h(x, y - 1) & \text{if } y \geq 1
\end{cases} \]

3. Suppose that \( f \) is a function whose values for certain argument pairs are presented in the chart below.

\[
\begin{array}{ccc}
f(0,0) = 5 & f(1,0) = 2 & f(2,0) = 0 \\
f(0,1) \text{ is undefined} & f(1,1) = 1 & f(2,1) = 2 \\
f(0,2) = 0 & f(1,2) = 0 & f(2,2) = 4 \\
f(0,3) \text{ is undefined} & f(1,3) = 3 & f(2,3) = 2 \\
f(0,4) = 10 & f(1,4) = 2 & f(2,4) = 0 \\
f(0,5) \text{ is undefined} & f(1,5) = 1 & f(2,5) \text{ is undefined}
\end{array}
\]

a) What is the value of \( \text{Mn}[f](0) \)?

b) What is the value of \( \text{Mn}[f](1) \)?

c) What is the value of \( \text{Mn}[f](2) \)?